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Chapter 2:
Algorithm Complexity Analysis

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- Why do we need to analyze the complexity of an algorithm?
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  - Algorithm complexity
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Reading

Cormen book:

Thomas, H. ,Cormen, Charles, E., Leiserson, and Ronald, L., Rivest . Introduction to Algorithms, The MIT Press.

(read Chapter 1 and 2, page 1-44).

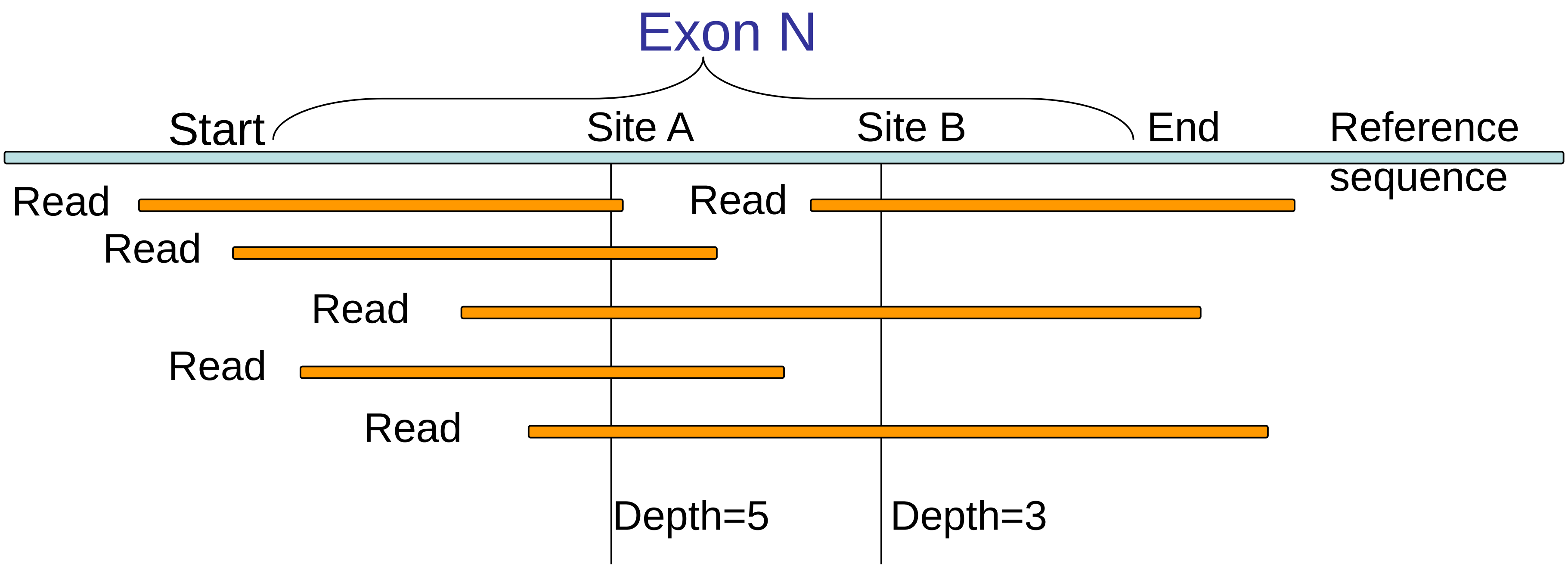
A real example: Exon-capture data analysis

There are ~60 millions of short reads sequenced from exon regions of a human genome. We need to figure out the how many exons were covered with at least 10 reads.

Steps:

1. Reads are aligned to the genome;
2. Each alignment is checked to see the exon it covers;
3. For each exon, check the number of reads cover the exon;
4. For all exons, filter out those with read number < 10.

A real example: Exon-capture data analysis



A real example: Exon-capture data analysis

1 days later

Student: I have created a program to do the analysis. It's running.

Teacher: Cool. Let me know when your analysis finishes.

A real example: Exon-capture data analysis

6 days later…

Student: My program has been running for 5 days, and it keeps on running. I have no idea about what is happening and what to do with it.

Teacher: Its core is a sorting algorithm with a complexity of at most O(N*lgN). It should be done within a few minutes!

Student: What?.....

Algorithm and its complexity

An **algorithm** is any well-defined computational procedure that takes in some **inputs** and produces some **outputs**.

Example: Sort an array of numbers

3, 2, 4, 5, 7, 1, 6 → 1, 2, 3,4, 5,6,7

Algorithm and its complexity

An algorithm is any well-defined computational procedure that takes in some inputs and produces some outputs.

Complexity: a function of **input size**

- Time complexity: the running time
- Space complexity: the memory size required

Algorithm and its complexity

Input size

- Number of items in the input
  - Sorting problem
  - FFT
- Total number of bits needed to represent the input
  - Arithmetic operation (+,-,x,/)
- The value of input
  - Factorial (N!)

Multiple input sizes

- Need to specify which input size is used
  - Graph operation (number of Vertices, and edges)

Algorithm and its complexity

Before we start

- we use a generic one-processor, random-access machine.
  - No parallel

Algorithm and its complexity

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

```
Insertion sort (A)
 for j = 2 to length(A)
       do key = A[j]
              /*insert A[j] into the sorted sequence A[1…j-1]
              i=j-1
               while i>0 and A[i]>key
                     do A[i+1]=A[i];
                           i=i-1;
                      A[i+1]=key;
```

Algorithm and its complexity

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

5	2	4	6	1	3
2	5	4	6	1	3
2	4	5	6	1	3
2	4	5	6	1	3
1	2	4	5	6	3
1	2	3	4	5	6

☺

Algorithm and its complexity

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

```
Insertion sort (A)
for j = 2 to length(A)
      do key = A[j]
            /*insert A[j] into the sorted sequence A[1…j-1]
            i=j-1
             while i>0 and A[i]>key
                  do A[i+1]=A[i];
                        i=i-1;
                  A[i+1]=key;
```


Algorithm time complexity: $O(N^2)$

Worst-case and average-case analysis

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

```
Insertion sort (A)
for j = 2 to length(A)
        do key = A[j]
                /*insert A[j] into the sorted sequence A[1…j-1]
                i=j-1
                 while i>0 and A[i]>key
                        do A[i+1]=A[i];
                           i=i-1;
                        A[i+1]=key;
```

Can repeat from 0 to j times

Algorithm time complexity: $O(N^2)$

Order of growth

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

Insertion sort:

Algorithm run time complexity: $O(N^2)$

Order of growth: 2

O-notation (big-O notation): Asymptotic upper bound

$O(g(n))$ = {$f(n)$: there exist positive constants c and n_0 such that $0 \le f(n) \le c\, g(n)$ for all $n \ge n_0$}

Note about O-notation operations:
$O(k_1 * N^2 + k_2 * N^3) = O(N^3)$ for constants k_1, k_2

O-notation (big-O notation): Asymptotic upper bound

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

Insertion sort:

algorithm time complexity: $O(N^2)$

Sorting with time complexity of O(N^2)

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

```
Sort (A)
for j = 2 to length(A)
    do key = A[j]
        /*Use binary search to insert A[j]
        /*into the sorted sequence A[1…j-1]
        i=j-1
        Binary_search(A[j], A[1…j-1],)
```

Sorting

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

There are a lot of sorting algorithms:

Heap sort (O(N*logN))

Merge sort (O(N*logN))

*Quick sort (worst-case O(N^2), average O(N*logN))

Merge sort

```
Merge-Sort (A, p, r)
        if p<r
                then q=[(p+r)/2]
                        Merge-Sort(A, p, q)
                        Merge-Sort(A,q+1,r)
                        Merge(A, p, q, r)
```

Time Complexity: $T(N)=\begin{cases} O(1); if\ N=1 \\ 2T(N/2)+O(N); if\ N>1 \end{cases}$

Solve it: T(N) = O(N*logN)

Space complexity

Example: Sort an array of numbers

5, 2, 4, 6, 1, 3 → 1, 2, 3,4, 5,6

Need an array of size N: A[1…N], and 3 temporary variables
O(N)

Example: Sequence alignment

Need a two-dimension array of size N*M, and a constant number of temporary variables
O(N*M) or O(max(N, M))

Other issues

- Input/Output method/place/mode
  - Speed
    - screen << hard disk << memory
- Programming language
  - Speed
    - Perl < java < C++ <C
- Output size
  - Blast: output can be a problem
  - Compressed data vs decompressed data
    - Smaller size
    - Higher read/write speed?